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Predicting Exhaust Plume Boundaries with Supersonic External Flows

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Several methods for predicting exhaust plume boundaries with a surrounding external flow currently exist. Unfortunately, these methods are usually cumbersome and often expensive, since they may be computationally intensive. Also, these methods typically provide many flowfield details in addition to the plume boundary location. If only the latter is desired, then calculation of these other details is wasted effort. This concern resulted in the development of a simplified plume boundary prediction method capable of analyzing underexpanded nozzle flow exhausting into a supersonic external flow. This new method is based upon the well-established Latvala method and uses an iterative scheme that employs two-dimensional flowfield assumptions. However, the method is still applicable to axisymmetric plumes, and its simplicity permits efficient operation on personal computers. Predictions of boundaries for axisymmetric plumes surrounded by various high-speed external flows exhibit excellent agreement with empirical data, and parametric studies indicate that trends are correctly predicted.

Nomenclature

A = area
M = Mach number
p = pressure
R = exit radius of nozzle
r = radial coordinate
x = axial coordinate
α = flow inclination angle
β = shock-wave angle
γ = ratio of specific heats
δ = incremental flow turning angle
θ = nozzle half-angle
Θ = average plume slope
ν = Prandtl-Meyer angle

Subscripts

ext = condition immediately outside of plume
n = downstream step position
noz = nozzle exit condition
t = stagnation quantity
∞ = freestream condition

Superscripts

$*$ = nozzle throat condition

Introduction

EXHAUST plume boundary prediction has many applications in the aerospace industry. In spacecraft design the extent of the exhaust plume is an important factor, since it may affect downlink communications. Also, an understanding of the interaction between the exhaust plume and various vehicle components, such as control surfaces, necessitates exhaust plume prediction for both aircraft and spacecraft. Another important application of exhaust plume prediction is in support of test programs making use of a freejet, which is an exhaust plume itself. In order to simulate in-flight aerodynamic performance, a freejet test facility must be configured so that desired Mach numbers, pressures, or other important parameters occur. Since these parameters are related to the freejet plume shape, calculations in advance are required in order to configure the test facility properly.

An exhaust plume expanding into a supersonic external stream further complicates the physics of the flowfield, as is illustrated in Fig. 1. The internal structure of the plume is composed of a system of reflected shocks that result in the familiar diamond-shaped exhaust plume pattern. Large gradients in flow parameters usually exist in this region. The supersonic external flow is also significantly affected as it encounters the exhaust plume. The initial interaction results in a shock, which allows the external flow to turn, adjusting to the presence of the plume. After the shock, the region where the two streams meet is characterized by a viscous-dominated shear layer. Clearly, any precise model of this flowfield would necessarily be complex and its solution difficult to obtain.

Often a simplified computational model that sacrifices some accuracy can be a useful tool if solutions become significantly easier to obtain. Such is the case with plume boundary prediction, particularly for preliminary design. However, a review of available techniques revealed that no simplified models for plume boundary prediction that incorporate supersonic external streams were available. Thus, an investigation was conducted to develop a simple numerical technique for the prediction of axisymmetric exhaust plume boundaries with a surrounding supersonic external stream.

Previous Work

Many computer simulations of exhaust plumes are currently available to predict boundaries for nozzles in quiescent environments. Codes based on the method of characteristics (MOC) have existed for years and are one example of the flowfield solvers available. MOC solutions normally provide detailed information throughout the exhaust plume, provided that appropriate boundary conditions can be specified. Unfortunately, the MOC approach is cumbersome in its complexity, particularly in three dimensions. Another possible solution approach is a finite-difference-based Navier-Stokes or parabolized Navier-Stokes technique. This, however, would likely involve an even greater amount of computational time and expense. If only general information is desired rather than a solution of the

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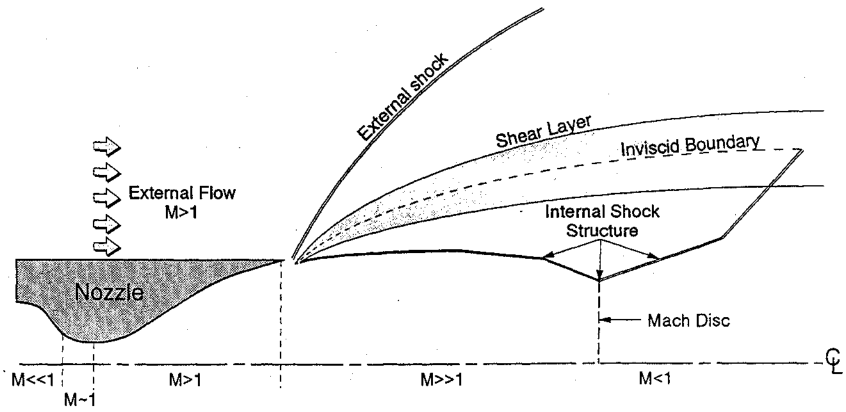

Fig. 1 Typical underexpanded plume.

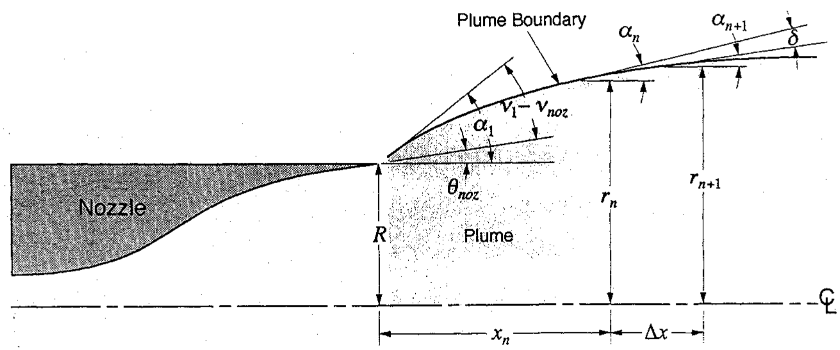

Fig. 2 The Latvala method.

entire flowfield, such complex approaches are not necessary. For example, if the location of the plume boundary is all that is needed, then the problem can be greatly simplified.

A number of approaches for approximating plume boundaries in quiescent environments have been proposed. Some are as simple as a circular-arc approximation that extends from the nozzle exit.[1] For a given nozzle exit Mach number and ratio of specific heats, approximations to the circular-arc radii are nearly invariant with pressure ratio in a region very near the nozzle exit. However, this boundary prediction technique is obviously an oversimplification and becomes inaccurate a short distance downstream from the nozzle exit plane. Another approach, based on isentropic expansion of the plume flowfield, was originally proposed by Latvala.[2] This numerical technique does not require as much time for calculation as a MOC or finite-difference approach, but is more accurate than the circular-arc approximation. With the Latvala method only plume boundary information is calculated, not any unwanted flowfield details.

Because of its widespread acceptance as a preliminary design tool, the Latvala method was used as a point of departure for the present study; thus a brief description of the theory behind the method is included here for completeness. (A complete description of the Latvala method may be found in Ref. 2.) The Latvala method is an extension of a method originally developed by Adamson and Nicholls.[3] If a very small section of a plume boundary is considered, as shown in Fig. 2, and radial flow from the nozzle is assumed, then the relationship between the geometric variables involved is given by

$$\frac{\Delta x}{r_n} = \left(\frac{r_{n+1}}{r_n} - 1\right) \cot\left(\alpha_n - \frac{\delta}{2}\right) \tag{1}$$

If the flow inclination angle α_n is known at point n on the plume boundary and an incremental turning angle $\delta = \alpha_n - \alpha_{n+1}$ is specified between point n and point $n + 1$ further downstream, the unknowns in Eq. (1) are $\Delta x / r_n$ and r_{n+1}/r_n.

For isentropic flow, the angle through which a flow turns is the difference between the Prandtl-Meyer angles at two points in the flow. Thus, if the Prandtl-Meyer angle is known at one point on the boundary, the Prandtl-Meyer angle at the adjacent point is equal to the original Prandtl-Meyer angle plus the angle through which the flow boundary turns, i.e., $\nu_{n+1} = \nu_n + \delta$. For each Prandtl-Meyer angle there is a corresponding area ratio, A/A^*. If the area ratio is considered to be a spherical area ratio (an implication of the radial flow assumption), then the radius ratio is given by

$$\frac{r_{n+1}}{r_n} = \sqrt{\frac{(A/A^*)_{n+1}(1 + \cos\alpha_{n+1})}{(A/A^*)_n(1 + \cos\alpha_n)}} \tag{2}$$

The values that must be specified to start the computations for the Latvala method are the nozzle exit Mach number M_{noz}; the ratio of specific heats, γ; the nozzle exit half-angle θ; and the nozzle pressure ratio $p_\infty / p_{t\,\text{noz}}$. The initial flow inclination is then given by

$$\alpha_1 = \nu_1 - \nu_{\text{noz}} + \theta \tag{3}$$

where ν_1 is the Prandtl-Meyer angle corresponding to the specified nozzle pressure ratio and ν_{noz} is the Prandtl-Meyer angle corresponding to the nozzle exit Mach number. An area ratio, A/A^*, that corresponds to ν_1 also can be calculated. Once α_1 and A/A^* are determined, the plume boundary point coordinates can be obtained using Eqs. (1) and (2). The remaining points are found by incrementing along the plume boundary with a specified incremental turning angle δ and the following relationships:

$$\nu_{n+1} = \nu_n + \delta \tag{4}$$

$$\alpha_{n+1} = \alpha_n - \delta \tag{5}$$

This process is repeated until the flow inclination angle is zero; the plume boundary location is assumed to be unchanging afterwards. However, comparison with empirical data has shown that to maintain accuracy within 5%, the Latvala method should be restricted to the first 6 nozzle radii downstream of the exit.[4]

Development of an Enhanced Prediction Method

The Latvala method was originally developed for the prediction of underexpanded exhaust plumes from axisymmetric nozzles in a

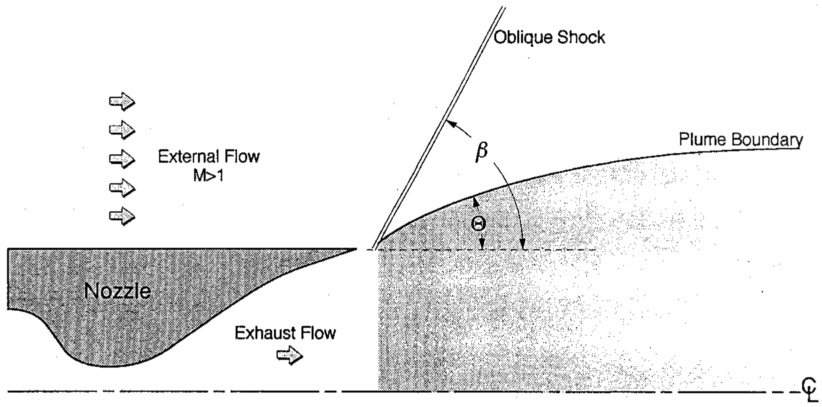

Fig. 3 Simplified flow model.

quiescent atmosphere. As such, it has become a well-established tool in the scientific community. The objective of the research presented here was to extend the capabilities of the Latvala method, making a well-established tool better. The problem at hand then becomes one of developing an approach that, while not significantly complicating the method, accounts for an external supersonic flow.

To solve this problem a simplification of the flowfield was used, an example of which can be seen in Fig. 3. The initial portion of the underexpanded exhaust plume can be modeled as a solid-boundary wedge, which turns the incoming supersonic external flow, thereby producing an oblique shock. Fundamental gasdynamic relationships reveal there is only one correct plume shape and oblique-shock relationship for a given set of flowfield conditions. The solution process, therefore, requires this correct relationship.

The Mach number of the external flow after the shock, the stagnation pressure loss across the shock, the flow deflection angle, and the shock-wave angle can all be found using oblique-shock theory. However, this process is iterative, since the shape of the plume dictates the strength of the shock, and the strength of the shock, in turn, will dictate a pressure that is impressed upon the plume, dictating its shape. In order to calculate the plume boundary in this situation, the Latvala method can be employed with a slight modification to the procedure. The pressure ratio used to start the calculations is now the ratio of the static pressure just outside the plume boundary to the nozzle stagnation pressure. This value is determined as follows:

$$\frac{p_{\text{ext}}}{p_{t\,\text{noz}}} = \frac{p_{\text{ext}}}{p_{t\,\text{ext}}}\frac{p_{\text{noz}}}{p_{t\,\text{noz}}}\frac{p_\infty}{p_{\text{noz}}}\frac{p_{t\,\text{ext}}}{p_{t\infty}}\frac{p_{t\infty}}{p_\infty} \tag{6}$$

where:

$p_{\text{ext}}/p_{t\,\text{ext}}$ is calculated from the isentropic relationship using M_{ext}.
$p_{\text{noz}}/p_{t\,\text{noz}}$ is calculated from isentropic relationship using M_{noz}.
p_∞/p_{noz} is given.
$p_{t\,\text{ext}}/p_{t\infty}$ is calculated from the oblique shock relationship.
$p_{t\infty}/p_\infty$ is calculated from the isentropic relationship.

The procedure for calculating the plume shape is as follows:

1) Using the Latvala method, calculate the plume using the given pressure ratio p_∞/p_{noz}.
2) Assume an average initial plume slope Θ.
3) Find the corresponding oblique shock angle β.
4) Calculate a new pressure ratio p_∞/p_{noz}.
5) Calculate a new plume using the Latvala method and the new pressure ratio.
6) Compare the new average plume slope with the previous value.
7) If ΔΘ is greater than the some acceptable deviation, iterate back to step 3.

This procedure is repeated until convergence upon the correct plume shape and oblique shock relationship is obtained. It is important to note that for this simplified flowfield, an average plume slope (or wedge angle) Θ is used to determine the correct oblique shock angle β. (The procedure for calculating the average plume slope is based on numerical experimentation and is discussed in detail in the next section.) By treating the plume as a solid boundary inclined at an angle Θ to the freestream flow moving at Mach number M_∞, simple oblique-shock theory yields the appropriate shock angle.

The oblique-shock method outlined is a simple approximation for a complex flowfield. However, it is possible to model the flowfield in another manner. In Fig. 2 it was shown how the plume was modeled as a solid boundary, or wedge, of infinite depth. In fact, the plume is not solid but is bounded by a loosely defined shear layer. The computational model developed above does not incorporate any aspect of the viscous effects of the shear layer. Furthermore, the actual plume is not of infinite depth; for an axisymmetric nozzle the plume is axisymmetric. The axisymmetry implies that there is a three-dimensional relieving effect as the supersonic external flow encounters the nozzle exhaust flow. The result is that the shock produced is actually closer to a conical shock, not an oblique shock.

For a given flow deflection angle, a conical shock is known to be weaker than an oblique shock,[5] and a weaker shock is accompanied by a smaller increase in static pressure. Since this pressure after the shock is a primary factor in the expansion of the exhaust plume, a conical shock is expected to cause greater plume spreading than an oblique shock. An alternative then is a plume prediction method based on the Latvala method but employing conical-shock theory rather than oblique-shock theory. Such a model intuitively should be more accurate, since it provides for the three-dimensional relieving effect that is known to occur in nature.

Clearly, a conical-shock method can be implemented in essentially the same way as the oblique-shock method presented above. The only difference is in the determination of the shock angle and the flow properties after the shock. However, this is readily accomplished by solving the Taylor-Maccoll equation[5] for conical flow. Once the freestream ratio of static pressure to nozzle exit pressure, p_∞/p_{noz}, is determined for the Latvala method (step 4 above), both solution techniques are identical.

Discussion of Results

The first phase of this investigation was to utilize the original Latvala method to develop a computer program for predicting exhaust plume expansion into a quiescent environment. Once validation of this program was completed, the external flow handling capability was added. The accuracy of the basic Latvala method implemented can be seen when it is compared with the experimental data of Love and Lee[6] in Fig. 4. This barely underexpanded case exhibits a slight underprediction of the plume boundary; however, the agreement is quite good, with a maximum error of less than 3% at $x/R = 6$. Results were not calculated further downstream, as the plume had reached maximum spreading, signifying that the Latvala method was no longer applicable.

The oblique-shock technique and the conical-shock technique were implemented separately in order to permit comparison of the two methods for handling a supersonic external flow. Both of these methods employed an average plume slope Θ that is related to a shock-wave angle β. The method for determining this average plume slope is of primary concern, since it dictates a corresponding shock-wave angle and thus can significantly affect the resulting plume. If only the first few plume boundary points (very near the nozzle

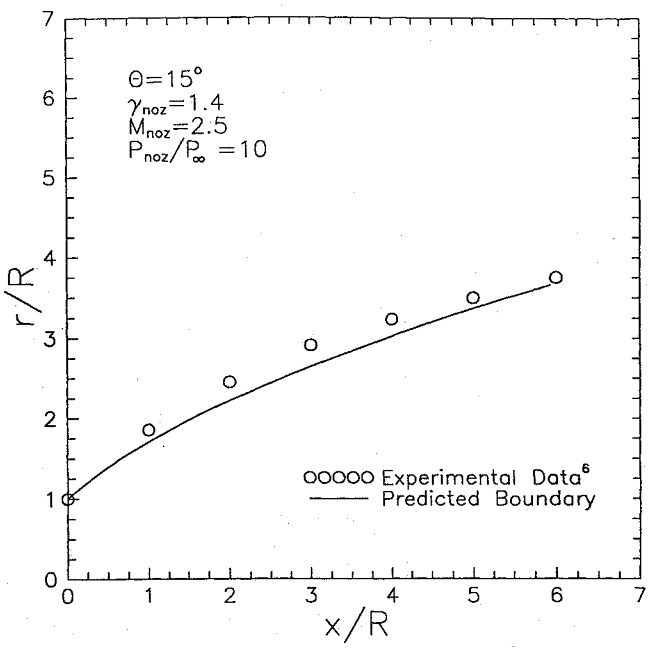

Fig. 4 Axisymmetric plume in a quiescent atmosphere.

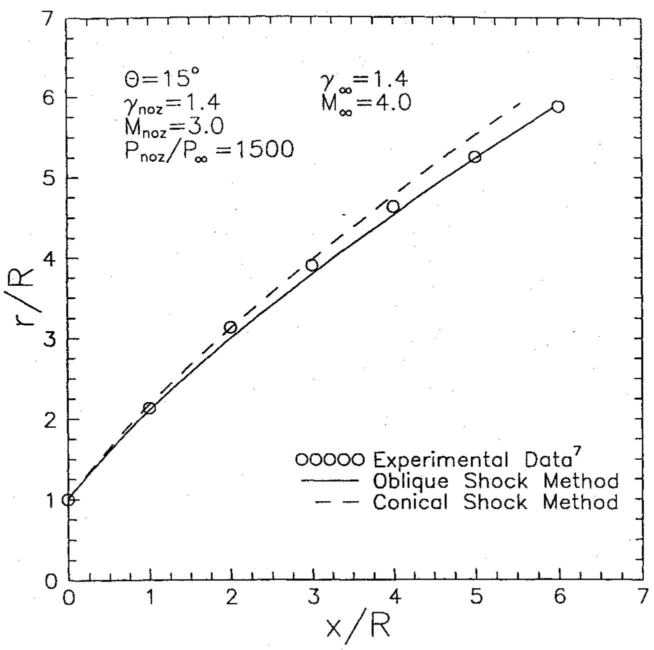

Fig. 5 Highly underexpanded plume in a supersonic external flow, $M_\infty = 4$.

exit) are used in determining the plume slope, the oblique shock produced is of high strength, resulting in a large rise in the external pressure. This pressure rise would in turn prevent the plume from expanding correctly downstream. Through numerical experimentation it was found that averaging the local plume slopes at eight locations ($x/R = 0.5, 1.0, 1.5, \dots, 4$) produced the best results. This approach extended the region for determining the shock over a reasonable portion of the initial plume, which intuitively is more representative, since the plume is not a true solid boundary. Another possible explanation of these results is that plume boundaries with similar pressure ratios tend to be similar in geometry near the nozzle exit. The differences occur further downstream, where the plume boundaries begin to approximate straight lines. If this more important (and larger) area is used to dictate the average plume slope, then better predictions are obtained.

Figures 5 and 6 represent highly underexpanded plumes exhausting into high-speed external flows. The flowfield parameters are

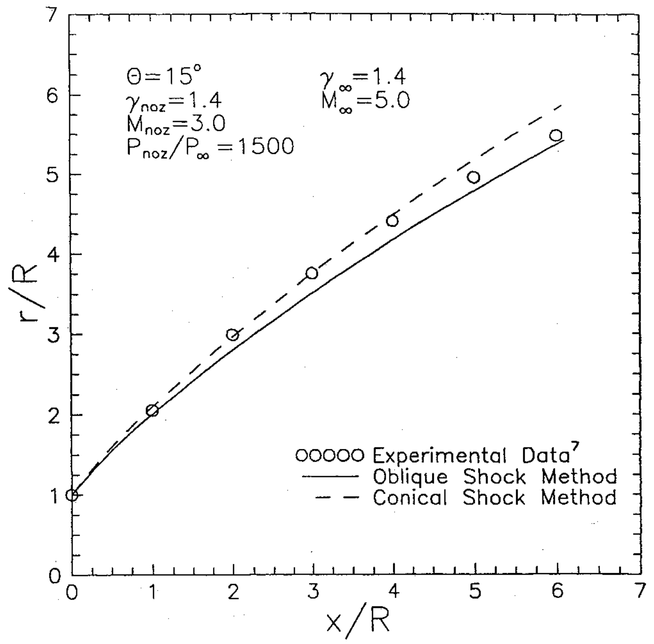

Fig. 6 Highly underexpanded plume in a supersonic external flow, $M_\infty = 5$.

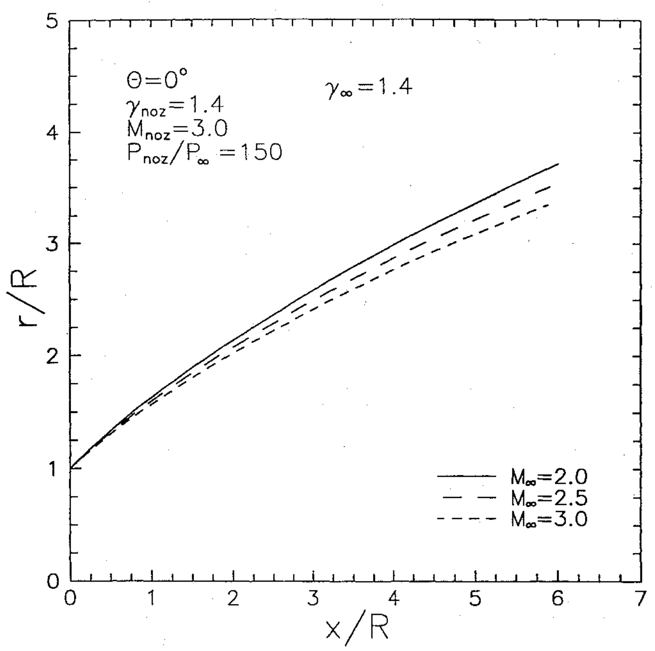

Fig. 7 Predicted plume boundaries with variations in M_∞.

identical in these two cases except for differing freestream Mach numbers of 4 and 5, respectively. The rapid plume expansion that occurred required large external-flow deflection angles. These required turning angles normally would cause an oblique shock to detach. In order to circumvent this problem and still obtain a solution, a replacement normal shock was extended from the nozzle lip into the external flow whenever conditions warranted the detachment of the oblique shock. The static pressure rise across this normal shock was calculated and used to determine a corresponding exhaust plume. The presence of the three-dimensional relieving effect apparently did not cause shock detachment to be a problem for the conical-shock method.

Both boundary prediction methods performed extremely well. Some divergence from the experimental data of Henson and Robertson[7] is seen to begin, particularly in Fig. 6, at $x/R = 6$. Comparison further downstream is not available because of the lack of experimental data. The conical-shock method provided a more

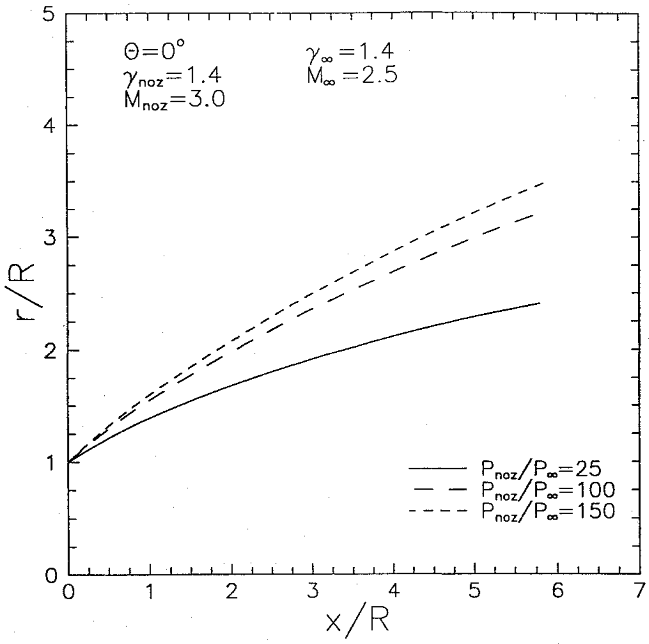

Fig. 8 Predicted plume boundaries with variations in pressure ratio.

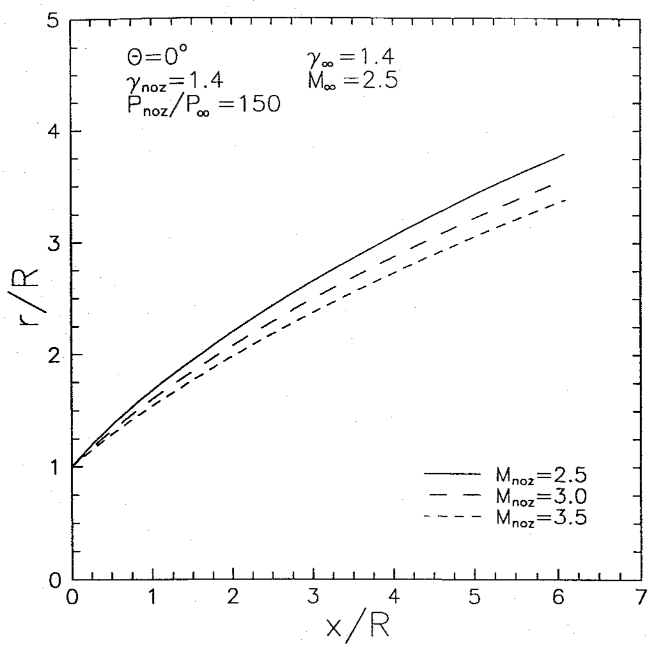

Fig. 9 Predicted plume boundaries with variations in nozzle exit Mach number.

conservative plume approximation and is actually nearly coincident with experimental data in the initial portion of the plumes.

An advantage of this new prediction technique is that plume boundaries are quickly generated, an important asset when performing parametric studies. To that end, some parametric studies were conducted to further validate the algorithms. Figures 7–10 show predicted plume boundaries for variations in nozzle flowfield parameters. The increase in jet spreading with increasing static pressure ratio, decreasing nozzle exit Mach number, decreasing specific-heat ratio, and decreasing freestream Mach number is apparent.

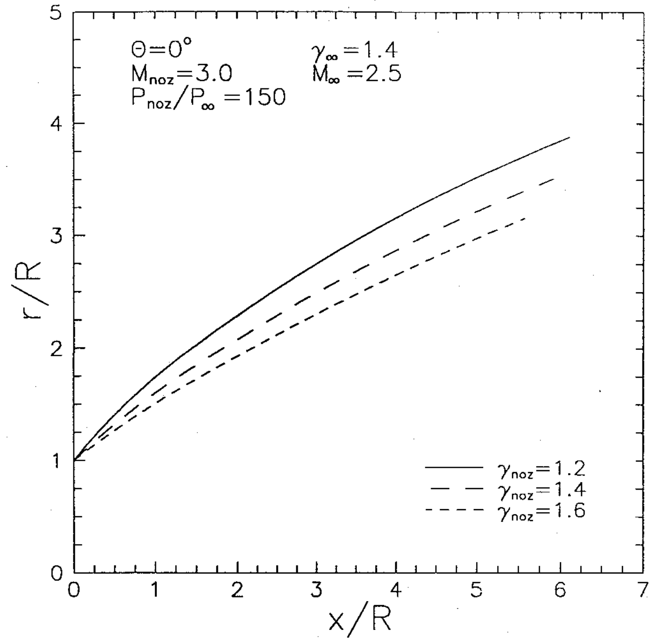

Fig. 10 Predicted plume boundaries with variations in ratio of specific heats.

Conclusions

A simplified plume boundary prediction method has been developed for applications where an underexpanded axisymmetric jet exhausts into a supersonic external flow. Results indicate excellent performance of the method for a range of Mach numbers and pressure ratios to approximately 6 nozzle exit radii downstream. Efforts are continuing to extend the range of the method to predict plume boundaries further downstream.

The prediction method developed actually contains two algorithms. It was found that the conical-shock algorithm did allow more plume spreading, but the difference from plumes predicted by the oblique-shock algorithm was very small. It would appear that the additional computational effort required to solve the conical-flow Taylor-Maccoll equation does not yield a significant advantage.

It should be noted that the accuracy of the Latvala method, which is the foundation of the present method, is remarkable considering its simplicity. Exhaust flow is highly irreversible, yet the Latvala method successfully employs isentropic expansion of the exhaust flow.

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